


$$\tan\left(\frac{315^\circ}{2}\right) = -\sqrt{\frac{1-\cos 315^\circ}{1+\cos 315^\circ}}$$


$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}$$

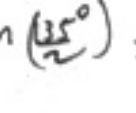
$$= -\sqrt{\frac{(2-\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}}$$

$$= -\sqrt{\frac{4-4\sqrt{2}+2}{4-2}}$$

$$= -\sqrt{\frac{6-4\sqrt{2}}{2}}$$

$$= -\sqrt{3-2\sqrt{2}}$$

13. $\tan 67.5^\circ$

$$\tan\left(\frac{135^\circ}{2}\right) = +\sqrt{\frac{1-\cos 135^\circ}{1+\cos 135^\circ}}$$


$$= +\sqrt{\frac{1-\frac{-\sqrt{2}}{2}}{1+\frac{-\sqrt{2}}{2}}}$$

$$= +\sqrt{\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2-\sqrt{2})}}$$

$$= +\sqrt{\frac{4+4\sqrt{2}+2}{4-2}}$$

$$= +\sqrt{\frac{6+4\sqrt{2}}{2}}$$

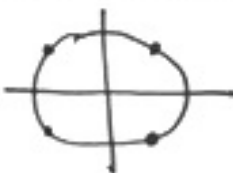
$$= +\sqrt{3+2\sqrt{2}}$$

Solve the following trigonometric equations. Use only the principal values.

14. $4\cos^2 x - 2 = 0$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{\sqrt{2}}{2}$$



$45^\circ, 135^\circ$

15. $\sin^2 x \csc x - 1 = 0$

$$\sin x - 1 = 0$$

$$\sin x = 1$$



90°

7.5

Solve the following trigonometric equations over the given domain.

16. $2\sin^2 x + 3\sin x - 2 = 0, 0 \leq x \leq 2\pi$

$$(2\sin x - 1)(\sin x + 2) = 0$$

$$\sin x = \frac{1}{2} \quad \text{and} \quad \sin x = -2$$



and

$30^\circ, 150^\circ$

$\frac{\pi}{6}, \frac{5\pi}{6}$

17. $\cos^2 x - 2\cos x - 3 = 0, 0^\circ \leq x \leq 360^\circ$

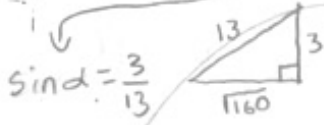
$$(\cos x + 1)(\cos x - 3) = 0$$

$$\cos x = -1 \quad \text{and} \quad \cos x = 3$$



180°

7. $\tan(\alpha + \beta)$ if $\csc \alpha = \frac{13}{3}$ and $\tan \beta = \frac{3}{4}$ if $0 < \alpha < \frac{\pi}{2}$ and $0 < \beta < \frac{\pi}{2}$.



$\sin \alpha = \frac{3}{13}$

If $\tan \theta = \frac{3}{4}$ and $180^\circ < \theta < 270^\circ$, find the exact value of each of the following.

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{\frac{3}{\sqrt{160}} + \frac{3}{4}}{1 - \frac{3}{\sqrt{160}} \cdot \frac{3}{4}}$$

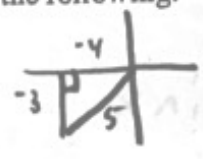
oops

$$\frac{(12 + 3\sqrt{160})}{(4\sqrt{160} - 9)} \cdot \frac{(4\sqrt{160} + 9)}{(4\sqrt{160} + 9)}$$

$$\frac{1920 + 108 + 48\sqrt{160} + 27\sqrt{160}}{2560 - 81}$$

$$\frac{2028 + 75\sqrt{160}}{2479}$$

$$\frac{2028 + 300\sqrt{10}}{2479}$$



8. $\sin 2\theta$

$$\sin 2\theta = 2 \cos \theta \sin \theta$$

$$= 2 \left(-\frac{4}{5}\right) \left(-\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

9. $\cos 2\theta$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$

$$\frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

10. $\tan 2\theta$

$$\frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{\frac{16 - 9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \cdot \frac{16}{7} = \frac{24}{7}$$

Use the half angle identities to find the exact value of the following function.

11. $\cos 22.5^\circ$

$$\cos \left(\frac{45^\circ}{2}\right) = + \sqrt{\frac{1 + \cos 45^\circ}{2}}$$

$$= + \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} \cdot \frac{2}{2}$$

$$= + \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$\frac{\sqrt{2 + \sqrt{2}}}{2}$$

Openers

Key

NAME _____

1.1 3 Simplify:

a. $\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$
 $\frac{1}{\cos^2 x}$

b. $\frac{\tan x}{\sin x}$

$\frac{\sin x}{\cos x} = \frac{\sin x}{\sin x} \cdot \frac{1}{\cos x}$

3a. $\frac{\sec^2 x}{}$

3b. $\frac{\sec x}{}$

1.2 4 Find a numerical value of one trigonometric function of each x.
 For example: $\cos x = 1$

a. $3 \cos x \csc x = 1$

$3 \cos x \frac{1}{\sin x} = 1$

$3 \cot x = 1$

b. $\cos x = \cot x$

$\left[\cos x = \frac{\cos x}{\sin x} \right] \cos x$
 $\cos x \sin x = \cos x$

4a. $\cot x = \frac{1}{3}$

4b. $\sin x = 1$

1.5 12 Solve: $2 \cos x \sin x = 3 \sin x$ for $0 \leq x < 2\pi$.

$2 \cos x \sin x - 3 \sin x = 0$

$\sin x (2 \cos x - 3) = 0$

$\sin x = 0$ $\cos x = \frac{3}{2}$
 und



12. $0, \pi$

1.5 13 Solve: $2 \sin^2 x = \sin x$ for principal values of x

$2 \sin^2 x - \sin x = 0$

$\sin x (2 \sin x - 1) = 0$

$\sin x = 0$ $\sin x = \frac{1}{2}$



13. $0^\circ, 30^\circ$

1.5 14 Solve: $(\tan x - 1)(2 \cos x + 1) = 0$ for all real values of x

$\tan x = 1$

$\cos x = -\frac{1}{2}$



14. $\frac{45^\circ + 180^\circ k}{120^\circ + 360^\circ k}$
 $\frac{240^\circ + 360^\circ k}{}$

Check your work!



Packet #2

Chapter 7 Common Assessment

Name _____

Simplify each of the following completely.

1. $\csc^2 \theta - \cot^2 \theta$

~~$\frac{1}{\sin^2 \theta} - \cot^2 \theta$~~

(1)

2. $\frac{\tan x}{\sin x}$

$\frac{\sin x}{\cos x} \div \sin x$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$
 $\frac{1}{\cos x}$

sec x

3. Find the value of $\cos x$ given $\tan x \csc x = 3$.

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$

$\frac{1}{\cos x} = 3$

$\cos x = \frac{1}{3}$ or $\sec x = 3$

Use the sum or difference identities to find the exact value of each function.

4. $\sin(225^\circ - 60^\circ)$

$\sin 225^\circ \cos 60^\circ - \cos 225^\circ \sin 60^\circ$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
 $-\frac{\sqrt{2}}{4} - \frac{-\sqrt{6}}{4} = \frac{-\sqrt{2} + \sqrt{6}}{4}$

5. $\cos(75^\circ)$

$(45^\circ + 30^\circ)$

$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$
 $\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

6. $\tan\left(\frac{11\pi}{12}\right)$

$\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{4}}$

$\frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$

$\frac{-\sqrt{3} + 1}{1 - \sqrt{3}}$

$\frac{-\sqrt{3} + 1}{1 - \sqrt{3}}$

$\frac{(-\sqrt{3} + 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$

$\frac{-3 + \sqrt{3} + \sqrt{3} - 1}{3 - 1}$

$\frac{-4 + 2\sqrt{3}}{2}$

$\frac{-4 + 2\sqrt{3}}{2}$

$\frac{-4 + 2\sqrt{3}}{2}$

$-2 + \sqrt{3}$

7.5 Practice NAME _____

Per _____
Student _____

1) SOLVE FOR $0 \leq x < 2\pi$

$$2 \sin^2 x + \sin x = 1$$

$$2 \sin^2 x + \sin x - 1 = 0$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2}$$

$$\sin x = -1$$



①. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

2) SOLVE FOR $0 \leq x < 2\pi$

$$\sin^2 x - \sin x + 1 = \cos^2 x$$

$$\sin^2 x - \sin x + 1 - \cos^2 x = 0$$

$$2 \sin^2 x - \sin x = 0$$

$$\sin x (2 \sin x - 1) = 0$$

$$\sin x = 0$$

$$\sin x = \frac{1}{2}$$



②. $0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

Packet #2

PKT #2

Practice 7.5

Section 7.5 Quiz
Solving Trigonometric Equations

Name: Key

1. Solve $2 \sin x + 2 = 0$ for $0 \leq x < 2\pi$.

$\sin x = -1$



1. $\frac{3\pi}{2}$

2. Solve $4 \cos^2 x - 3 = 0$ for principal values of x .



2. $30^\circ, 150^\circ$

3. Solve $\tan x - 1 = 0$ for all real values of x .

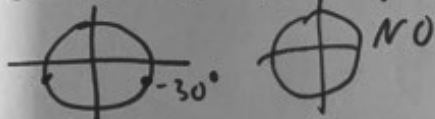
$\tan x = 1$



3. $45^\circ + 180^\circ K$

4. Solve $2 \sin^2 x - 5 \sin x + 2 = 0$ for principal values.

$(2 \sin x + 1)(\sin x - 2) = 0$



4. -30°

5. Solve $4 \sec x \tan x = 5 \tan x$ for $0^\circ \leq x < 360^\circ$.

$4 \sec x \tan x - 5 \tan x = 0$

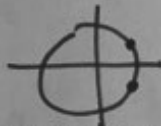
$\tan x (4 \sec x - 5) = 0$

$\tan x = 0$



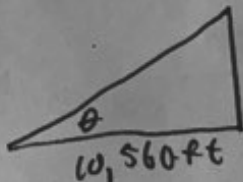
$\sec x = 5/4$

$\cos x = 4/5$



5. $0^\circ, 180^\circ, 38.66^\circ, 321.36^\circ$

6. An airplane takes off from the ground and reaches a height of 500 feet after flying 2 miles. Given the formula $H = d \tan \theta$, where H is the height of the plane and d is the distance (along the ground) the plane has flown, find the angle of ascent θ at which the plane took off.



500 ft

10,560 ft

$\tan \theta = \frac{500}{10,560}$

$\theta = 2.71^\circ$

$\theta = \tan^{-1} \left(\frac{500}{10,560} \right)$