

Practice

Powers and Roots of Complex Numbers

Find each power. Express the result in rectangular form.

1. $(-2 - 2\sqrt{3}i)^3$
64

2. $(1 - i)^5$
 $-4 + 4i$

3. $(-1 + \sqrt{3}i)^{12}$
4096

4. $\left[1\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^{-3}$
 $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

5. $(2 + 3i)^6$
 $2035 - 828i$

6. $(1 + i)^8$
16

Find each principal root. Express the result in the form $a + bi$ with a and b rounded to the nearest hundredth.

7. $(-27i)^{\frac{1}{3}}$
 $2.60 - 1.5i$

8. $(8 - 8i)^{\frac{1}{3}}$
 $2.17 - 0.58i$

9. $\sqrt[5]{-243i}$
 $2.85 - 0.93i$

10. $(-i)^{\frac{1}{3}}$
 $0.87 + 0.5i$

11. $\sqrt[8]{-8i}$
 $1.27 - 0.25i$

12. $\sqrt[4]{-2 - 2\sqrt{3}i}$
 $1.22 - 0.71i$

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Algebraic Nu

A complex number is a polynomial with integer coefficients where the exponents are integers with no common factor of $qx - p$. This shows that irrational numbers are algebraic.

Example Show that $\sqrt{2}$ is algebraic.

Let $x = \sqrt{2}$.

$$x^2 = 2$$

$$(x^2 - 2) = 0$$

$$x^2 - 2 = 0$$

$$x^2 - 2x^2 = 0$$

$$x^2 - 2x^2 = 0$$

Thus, $\sqrt{2}$ is algebraic.

algebraic

If a complex number is a root of a polynomial with integer coefficients, then the number is algebraic. The best-known transcendental numbers are e and π . These numbers are not algebraic until 1873 that the mathematician Charles Hermite was able to show that e is transcendental. C. L. F. Lindemann

Show that each complex number is a zero of a polynomial with integer coefficients.

1. $\sqrt{2}$
 $x^2 - 2$

3. $2 - i$
 $x^2 - 4x + 5$

5. $4 - \sqrt[4]{2}i$
 $x^4 - 16x^3 + 9$

7. $\sqrt{1 + \sqrt[3]{5}}$
 $x^6 - 3x^4 + 3$