

## Practice

## Products and Quotients of Complex Numbers in Polar Form

Find each product or quotient. Express the result in rectangular form.

$$1. 3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \cdot 3\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$$

**9**

$$2. 6\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \div 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

**$\frac{3\sqrt{3}}{2} + \frac{3}{2}i$**

$$3. 14\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) \div 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

**$-\frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$**

$$4. 3\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) \cdot 6\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

**$-9\sqrt{3} - 9i$**

$$5. 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \cdot 2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$$

**$2\sqrt{3} - 2i$**

$$6. 15(\cos \pi + i \sin \pi) \div 3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

**$5i$**

7. **Electricity** Find the current in a circuit with a voltage of 12 volts and an impedance of  $2 - 4j$  ohms. Use the formula,  $E = I \cdot Z$ , where  $E$  is the voltage measured in volts,  $I$  is the current measured in amperes, and  $Z$  is the impedance measured in ohms.

(Hint: Electrical engineers use  $j$  as the imaginary unit, so they write complex numbers in the form  $a + bj$ . Express each number in polar form, substitute values into the formula, and then express the current in rectangular form.)

**$1.2 + 2.4j$  amps**

## Complex C

In Lesson 9-5, you saw that the complex conjugates of  $a + bi$  and  $a - bi$  are  $a - bi$  and  $a + bi$ , respectively. The product of a complex number and its conjugate is a real number.

1. Show that the square of a complex number is a real number. **The solution is  $a = -1$  and  $b = 0$ .**

2. Show that the discriminant of a quadratic equation with complex coefficients is a real number. **By the quadratic formula,**

$$-\frac{B}{2A} + i \frac{\sqrt{B^2 - 4AC}}{2A}$$

**so  $a = -\frac{B}{2A}$**

**The conjugate of**

3.  $z = a + bi$ . Use the formula for the conjugate of a complex number:

$$\frac{a - bi}{a^2 + b^2}$$

4.  $z = r(\cos \theta + i \sin \theta)$ . Use the formula for the conjugate of a complex number:

$$r[\cos(-\theta) - i \sin(-\theta)]$$

**Use your answer to**

5. Find  $z \cdot \bar{z}$ .

$$r^2 = |z|^2$$

6. Find  $z \div \bar{z}$ . ( $z = r(\cos \theta + i \sin \theta)$ )

$$\cos 2\theta + i \sin 2\theta$$